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Optimizing Species Composition in Uneven-Aged Forest Stands

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ABSTRACT. This article describes an approach for determining the optimal sustainable equilibrium diameter distribution and species composition for a mixed-species forest stand. Using the Prognosis Model–a single tree distance independent growth model and its attendant regeneration subsystem—the maximization objective is formulated in terms of three decision variables per species: (1) the scale and shape parameters of a Weibull distribution function, and (2) the total number of trees per acre. A direct search, derivative-free, constrained nonlinear programming algorithm is used to optimize the growth model under a sustainable equilibrium constraint. To facilitate optimization, the stochastic features of the Prognosis Model are transformed to their deterministic counterparts. Results are presented for the *Abies lasiocarpa/Clintonia uniflora* habitat type found in northern Idaho. FOR. SCI. 33(4):958–970.

KEYWORDS: Diameter distributions, Weibull distribution function, Prognosis Model, Investment efficiency.

OPTIMIZING DECISIONS FOR UNEVEN-AGED STANDS involves determination of the following major questions: (1) the optimal sustainable diameter distribution including maximum tree size, level of growing stock, and distribution of trees by diameter class, (2) the optimal species mix, (3) the optimal cutting cycle, (4) the optimal conversion strategy and length, and (5) the optimal schedule of treatments for all stands to best meet forestwide objectives and constraints (Hann and Bare 1979). Previously published work has concentrated on the first and third of these questions, but researchers are beginning to address the remaining questions using sophisticated tools of operations research.

Since publication of Adams (1974) and Adams and Ek's (1974) pioneering studies involving a mixed-species northern hardwood stand, much work related to the major questions listed above has been performed. Adams (1976) defined and developed "investment-efficient" sustainable equilibrium diameter distributions that maximized a marginal value growth percent criterion (equivalent to maximizing land expectation value) originally cited by Duerr and Bond (1952). Buongiorno and Michie (1980) and Michie (1985) used a fixed coefficient matrix model for the same mixed-species northern hardwood stand to develop both sustainable equilibrium diameter distributions and conversion strategies that maximized the land expectation value. Harrison and Michie (1985) extended the matrix approach by allowing growth projections over intervals shorter than the growth period. Solomon et al. (1986) further extended the matrix approach by relating transition probabilities to tree size and density for mixed-species stands of New England.

Martin (1982) also determined sustainable equilibrium diameter distributions for northern hardwoods by maximizing the land expectation value. However, unlike Adams and Ek (1974), who characterized sustainable equi-

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librium diameter distributions using diameter classes, Martin used a Weibull distribution function, thus reducing the decision space to three variables (i.e., the number of trees and the two parameters of the Weibull distribution). After preliminary analysis, the shape parameter was set to one, thus reducing the decision space to two variables.

Hansen and Nyland (1987) demonstrated the interrelationship between maximum tree size, level of growing stock, cutting cycle, and management objectives for uneven-aged stands of sugar maple (*Acer saccharum*). Simulated results were shown for a variety of scenarios based on constant and mixed "q" ratios.[1]

Lynch and Moser (1986) developed a system of first-order ordinary differential equations for mixed-species stands involving two species groups. Their whole stand model included parameter recovery procedures for the Weibull diameter distribution function (Hyink and Moser 1983), but no attempt at stand optimization was reported.

In addition to examining the first and third questions, Chang (1981) addressed a constrained version of the stand conversion problem and concluded that maximization of the land expectation value was equivalent to maximization of the forest value (land plus growing stock). Using comparative statics, but ignoring diameter class dynamics, Chang determined sustainable equilibrium levels of growing stock simultaneously with the optimal cutting cycle. His consideration of conversion strategies was limited to the *special* case where the sustainable equilibium level of residual growing stock was to be reached in one harvest, assuming that the initial stand contained sufficient growing stock to allow this harvest.

Hall (1983) showed how to determine the optimal level of growing stock and cutting cycle to maximize the land expectation value. As with Chang, no diameter class dynamics were considered. However, unlike Chang, Hall began his analysis immediately after harvest to the optimal sustainable equilibrium level of growing stock. Hall and Bruna (1983) developed a simulation model that allowed users to experiment with a wide variety of management decisions to winnow out the preferred sustainable equilibrium diameter distribution that maximized the land expectation value. Stand growth dynamics were built into the simulator by using the constant q of de Liocourt, implying a balanced diameter distribution. This assumption was also employed by Martin (1982).

Rideout (1985) discussed the optimization of decisions in uneven-aged stands in the absence of diameter class dynamics and argued that the appropriate financial criterion for determining the optimal steady-state level of growing stock was the "managed forest value"—the present value of a perpetual series of equal harvest incomes. This differs from Adams (1976), Buongiorno and Michie (1980), Michie (1985), and Martin (1982), who maximized the land expectation value when determining optimal steady-state diameter distibutions. As discussed by Haight (1987), and demonstrated later in this paper, the two criteria do not produce consistent results when answering the steady-state stand structure question (i.e., question one). Lyon (1983) and Nautiyal (1983) also analyzed the economics of uneven-aged management using a comparative statics approach.

Rapera (1980) formulated and solved an optimal control theoretic model

[1] These ratios express the number of trees in a given diameter class to the number in the next larger class.

for northern hardwoods that solved for the optimal conversion strategy without specifying the form of the terminal diameter distribution [a procedure previously suggested by Adams and Ek (1975)]. In maximizing the present value of net returns over a fixed planning horizon, he found that many locally optimal solutions existed.

Bullard et al. (1985) developed a nonlinear-integer programming model to optimize the species mix and diameter distribution of a northern hardwood stand. However, by omitting the ingrowth equation of Adams and Ek (1974), they simulated the optimal thinning and rotation decisions for even-aged management on a species/diameter class basis. Because their nonlinear-integer programming formulation defied exact numerical solution, they used a heuristic random search algorithm to obtain approximate solutions to problems involving two species and up to four diameter classes. Roise (1986) used a direct search algorithm in optimizing even-aged management decisions for a single species model. Valsta (1986) used dynamic programming to optimize thinning/rotation decisions in even-aged pine-birch stands, where birch percentage of volume was one of three state variables.

Haight et al. (1985) and Haight (1985) utilized a control theory formulation to reexamine the optimization of the Adams and Ek (1974) northern hardwood growth model. Haight et al. (1985) expanded the dimensions of the problem by concentrating on the joint optimization of steady-state stand structure and conversion strategy (i.e., questions one, three, and four) and solved the same problem as Rapera (1980) using a different solution procedure. Their objective function was to maximize the present value of net returns over a finite planning horizon of 150 years without regard to the form of the terminal diameter distribution or level of intermediate harvests. This objective "can be viewed as the present value of future income that could be obtained by managing the existing land and timber . . . indefinitely" (Haight et al. 1985). Thus, maximization of the land expectation value, with its accompanying "investment-efficient" diameter distribution, was not the objective of the analysis.

In comparing the efficiency of even-aged and uneven-aged stand management, Haight (1987) utilized Hann's (1980) ponderosa pine growth model to again conclude that maximization of the land expectation value in the presence of an equilibrium sustainability constraint does not optimize the present value of net returns over a finite planning horizon. However, his derived steady-state diameter distributions do not maximize the land expectation value.

Based on a review of these studies, the following observations are offered:

1. Most stand-level optimization work published to date utilizes Adams and Ek's (1974) whole stand mixed-species northern hardwood growth model where individual species dynamics are not recognized. Published results of species composition optimization or the optimization of distance independent individual tree growth models for uneven-aged stands could not be found.
2. Most studies use a static analysis to determine the optimal sustainable equilibrium diameter distribution (or level of growing stock) that maximizes the land expectation value and do not rely on the assumption of a constant q ratio. At least one author (Rideout 1985) advocates maximization of the managed forest value for this purpose. These analyses address the cutting cycle and steady-state stand structure questions.
3. Several papers have addressed the derivation of dynamic solutions that answer the conversion strategy question. For comparison, these analyses have started

with an initial stand equivalent in structure to the optimal sustainable equilibrium "investment-efficient" solution found by maximizing the land expectation value. These studies maximize the present value of net returns and lead to steady-state solutions with lower land expectation values than obtained from the static analyses. However, they do not require that a constant cutting cycle be used (although harvests can be made no more often than each cycle); they place no restrictions on the form of the terminal diameter distribution; and they do not ensure that a constant harvest be maintained during, or at the end of, the finite planning horizon. Predictably, the present values associated with these solutions exceed those of the more constrained static analyses, but the terminal distribution is not "investment-efficient."

OBJECTIVES

The objectives of this paper are to determine the optimal sustainable equilibrium diameter distribution and species mix for a distance independent individual tree growth model. In addition to answering questions one and two (the equilibrium stand structure and species mix questions), the optimization model also addresses question three (the cutting cycle question). However, the cutting cycle is treated as a model parameter and not a decision variable. No attempt is made to address the conversion strategy or forest-wide scheduling questions. Throughout the analysis, maximization of the land expectation value, managed forest value, and board foot volume growth are used for comparative purposes. However, the land expectation value is the appropriate criterion for determining sustainable equilibrium diameter distributions if economic efficiency is the objective of management.

MODEL OVERVIEW

For purposes of illustration, the Prognosis Model (Wykoff et al. 1982, Wykoff 1986) is used because it meets the two criteria of: (1) single tree distance independence, and (2) species dependent growth dynamics. For optimizing this growth model, the "complex" method of Box (Kuester and Mize 1973) is employed. This constrained nonlinear programming algorithm is a derivative-free, direct search technique that utilizes an initial set of solutions scattered throughout the decision space. As solutions are found that improve the value of the objective function, the algorithm climbs toward a locally optimum solution. A global solution is not guaranteed but, by running the optimization model several times with different sets of initial solutions scattered across the decision space, a thorough examination of the decision space can be conducted.

To reduce the number of decision variables, it is assumed that the sustainable equilibrium diameter distribution can be modeled by a Weibull distribution function [see Martin (1982)]. This allows the optimization problem to be formulated in terms of three decision variables per species: (1) the scale and shape parameters of the Weibull distribution, and (2) the total number of trees per acre. This greatly reduces the size of the optimization problem as compared to the control theory formulations reviewed above. However, this assumption implies that the optimal sustainable equilibrium diameter distribution can be adequately represented by a continuous, unimodal function. Although not presented here, this assumption can be investigated empirically by comparing results with those produced by other modeling approaches (e.g., a diameter class model).

The general form of the stand-level optimization problem is:

$$MAX\ LEV_t = VG_t/((1 + i)^t - 1) - VGS_t$$

or

$$MAX\ MFV_t = VG_t + VG_t/((1 + i)^t - 1)$$

or

$$MAX\ BFG_t$$

Subject to

$$X'_{ds} >= X_{ds}$$

for all $d = 1, 2, \ldots, M$ and $s = 1, 2, \ldots, K$ (Equilibrium – sustainability)

$$B_s > 0,\ C_s > 0 \text{ and } N_s >= 0$$

for all $s = 1, 2, \ldots, K$ (Nonnegativity)

The valuation formulae utilized in the above equations are:

$$VGS_t = \sum_{s=1}^{K} \sum_{u=1}^{N_s} P_{su} V_{su} R_{su}$$

$$VG_t = \sum_{s=1}^{K} \sum_{u=1}^{N'_s} P_{su} V_{su} R_{su} - VGS_t$$

Where

LEV_t = Per acre land expectation value for t-year cutting cycle

MFV_t = Per-acre managed forest value (land and trees) for t-year cutting cycle

BFG_t = Per-acre board foot volume growth harvested every t-years

VG_t = Per-acre value growth harvested every t years

VGS_t = Per-acre value of the residual growing stock for t-year cutting cycle

i = Real rate of interest

t = Cutting cycle length ($t = 10\ NGP$)

NGP = Number of ten-year growth projection periods in cutting cycle

X_{ds}, X'_{ds} = Number of trees in dth diameter class for sth species at beginning and end of cutting cycle, respectively.

M = Number of diameter classes

K = Number of species

P_{su} = Stumpage price ($/mbf Scribner) for uth tree, sth species

V_{su} = Scribner board foot volume for uth tree, sth species

R_{su} = Survival tree factor for uth tree, sth species. This is the number of trees per acre represented by the uth tree, sth species.

N_s,N'_s = Total number of trees per acre of sth species at beginning and end of cutting cycle, respectively.

B_s = Weibull distribution scale parameter for sth species

C_s = Weibull distribution shape parameter for sth species

The expression for VG_t represents the difference between the value of the growing stock at the beginning and end of the cutting cycle—computed using individual trees. By setting all stumpage prices (P_{su}) equal to one, a comparable formula for BFG_t (board foot volume growth) is obtained. The purpose of the optimization is to select a steady-state structure that maximizes the chosen objective function while satisfying the equilibrium sustainability constraint.

The width of the diameter class chosen for this constraint (3 in. in this paper) can significantly affect the structure of the optimal steady-state solution. And, as class width increases, the potential effect of the constraint decreases in magnitude. Most published stand optimization studies are based on diameter class growth models and adopt the class width used in the growth model. However, for individual tree growth models, the diameter class width used in the equilibrium sustainability constraint becomes a parameter set by the analyst.

Costs or returns associated with either intermediate or annual activities are easily incorporated into the appropriate financial objective function. However, with the exception of a precommercial thinning, no such activities are included in the results presented here.

As stated above, the maximization is carried out over three decision variables per species—B_s, C_s, and N_s. The relationship between X_{ds} and the decision variables is shown in the following equation (Martin 1982, Bailey and Dell 1973):

$$X_{ds} = \frac{N_s\left\{\exp\left[-\left(\frac{DL_{ds}}{B_s}\right)^{C_s}\right] - \exp\left[-\left(\frac{DU_{ds}}{B_s}\right)^{C_s}\right]\right\}}{1 - \exp\left[-\left(\frac{MD}{B_s}\right)^{C_s}\right]}$$

In this equation MD, DL_{ds}, and DU_{ds} are, respectively, the diameter of the largest tree allowed in the residual stand and the lower and upper diameter limits for the dth diameter class and sth species.

For all results presented here, the maximum tree is limited to 27 in. in diameter. Although this was not a significant factor in the results reported below, other maximum tree size parameters might produce different results. After N_s trees are assigned to 3-in. diameter classes (for a given set of values for B_s and C_s), a beginning tree list is generated by assuming that trees are uniformly distributed within each diameter class. The resulting initial distribution of trees approximates a Weibull distribution.

GROWTH MODEL DEVELOPMENT

The deterministic version of the Prognosis Model developed for purposes of optimization consists of three parts: (1) small and large tree diameter increment functions, (2) mortality functions, and (3) regeneration functions. The diameter increment functions are taken from Wykoff (1986); the mortality functions are taken from Wykoff et al. (1982) and Wykoff (1986); and the regeneration functions are derived from Ferguson et al. (1986) and the FORTRAN source code for version 5.0 of the Prognosis Model.

Given a beginning tree list (made up of one or more species), the deterministic growth model first uses the ten-year diameter increment functions to update the tree diameters. The mortality functions are then employed to compute ten-year survival proportions. Finally, trees are added to the tree list by the regeneration subsystem, and Scribner board foot volumes are calculated to facilitate evaluation of the chosen objective function. The model repeats this cycle as many times as specified by the user to account for the cutting cycle of interest. A more detailed description of the deterministic version of the Prognosis Model can be found in Bare and Opalach (1987).

The first step in the optimization process is to construct the initial complex. This is a set of points in decision space, which consists of 3*K dimensions—where K is the number of species being considered. Each point in the complex has an associated objective function value. Initially, the points in the complex are generated to ensure broad coverage of the decision space. For each point in the initial complex, a tree list is generated for the selected values of N_s, C_s, and B_s—the 3*K decision variables. This tree list is passed to the deterministic growth model, and an updated tree list is returned. After evaluating the objective function, the equilibrium sustainability constraint is checked, and the objective function value of a solution that violates the constraint is severely reduced by use of a penalty function.

Given the initial complex, the algorithm begins an iterative process to locate the optimum value of the objective function in the decision space defined by the initial complex. The algorithm determines which point in the complex has the lowest objective function value, and omitting this point, it computes the centroid of the remaining points. The lowest point and the centroid are then used to define the search direction. A new point is located in this direction and the growth model is used to obtain a new tree list and associated value of the objective function. This iterative process continues until convergence criteria are satisfied, or until the number of iterations exceeds a user specified maximum. A flowchart of this process is shown in Figure 1.

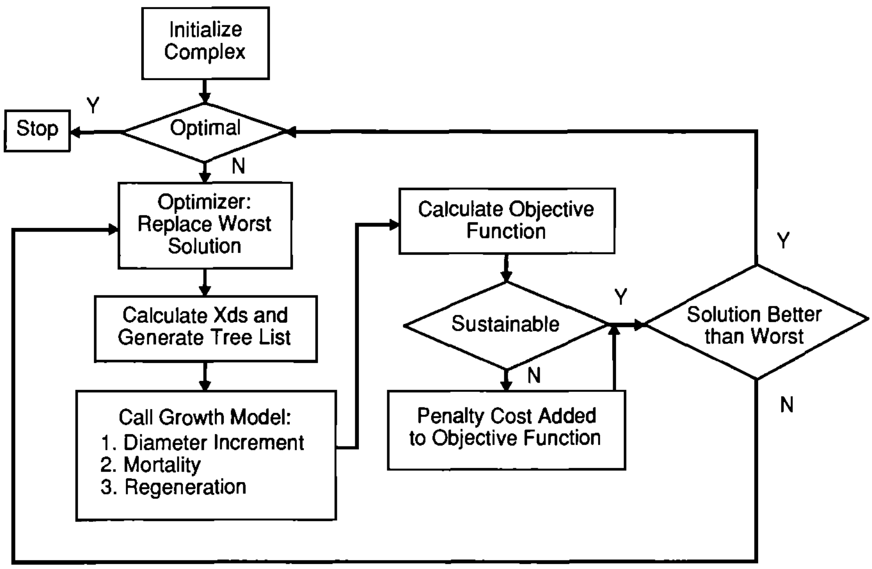

FIGURE 1. Flowchart of model operation.

MODEL EXPERIMENTATION

In order to demonstrate the above procedure, the deterministic growth model and optimization algorithm are applied to the *Abies lasiocarpa/Clintonia uniflora* habitat type found on the Coeur d'Alene National Forest in northern Idaho. This is a common type in the Rocky Mountains and typically consists of *Abies lasiocarpa, Picea engelmannii, Pseudotsuga menziesii, Larix occidentalis, Pinus contorta,* and *Pinus monticola* and occurs from the lower mountain valleys at about 3,000 ft up to about 6,000 ft elevation. Results reported below assume an elevation of 4,500 ft, a slope of 10%, and an aspect of 0°.

While results for a ten-year cutting cycle are reported, other period lengths can easily be tested. And, for demonstration purposes, the derived sustainable equilibrium distributions are restricted to at most two species, *Abies lasiocarpa* and *Picea engelmannii.* However, the model is capable of handling more than two species. Other species added to the tree list by the regeneration system are considered "undesirable" and are removed in a precommercial thinking.

A 5% real interest rate is used when maximizing managed forest value and land expectation value. Stumpage prices ($/mbf Scribner) are a function of species and tree diameter, but only trees greater than 9.0 in. have positive stumpage values (see Table 1). It is assumed that trees smaller than this are removed in a precommercial thinning conducted once per cutting cycle. Two cost scenarios are examined: (1) a "zero" cost alternative where the cost of the precommercial thinning is covered by the sale of the material thinned, and (2) a "true" precommercial thinning conducted at the rate of 0.08, 0.15 and 0.25 ($/tree) for trees in the 0–3, 3–6, and 6–9 in. diameter classes, respectively.

Readers interested in the silvicultural characteristics of *Abies lasiocarpa* or *Picea engelmannii,* two common species which occur in the *Abies lasiocarpa/Clintonia uniflora* habitat type, are encouraged to refer to Alexander et al. (1984) or Alexander and Shepperd (1984), respectively. While both even-aged and uneven-aged silvicultural systems can be used in this type, watershed, recreational, wildlife, and amenity objectives often favor the latter system (Alexander and Edminster 1977).

Results of the optimization are given in Tables 2–4, where estimates for N_s, B_s, and C_s plus other descriptive statistics are shown for each species.

TABLE 1. Tree values and board foot volumes used in model experimentation.

Diameter	*Picea*		*Abies*	
(in.)	$/tree	bf/tree	$/tree	bf/tree
9	0.00	71.7	0.00	60.0
11	1.78	113.6	0.93	93.1
13	4.83	167.9	2.50	135.5
15	9.38	234.7	4.80	187.6
17	15.64	314.2	7.94	249.2
19	23.78	406.4	12.00	320.6
21	33.51	505.2	16.65	392.4
23	45.85	624.1	21.37	454.8
25	60.47	756.1	26.80	523.9
27	77.50	901.0	32.99	599.7
29	97.02	1059.0	39.98	682.3

Source: Derived from average stumpage prices for sawtimber sold on National Forests, Northern Region, 1974–84.

Table 2 shows the "investment-efficient" equilibrium sustainable diameter distribution that maximizes the land expectation value for the "zero" cost precommercial thinning scenario. One striking result is the low level of residual growing stock (630 bf for trees ≥9.0 in. in diameter) associated with this solution. However, the 10-yr volume growth of 6696 bf results in a land expectation value of $256/ac at 5% interest. The 196 trees/ac carried in the residual stand are valued at $16/ac and consist almost exclusively of *Picea*. However, this is only achieved if 262 *Picea* and *Abies* plus 474 trees (not shown in Table 2) of other species (all less than 9 in. in diameter) are removed every 10 years.

A second interesting result shown in Table 2 is that *Abies* is essentially extinguished from the optimal residual diameter distribution. As explained below, this is due to inherent characteristics of the Prognosis Model and the assumed stumpage prices which favor *Picea*.

The shape parameter (C_s) for *Picea* is approximately equal to 1—indicating that the negative exponential function describes the distribution of trees by diameter class. Thus the residual diameter distribution is balanced and sustainable over a 10-yr cutting cycle. No trees larger than 24 in. are present in the optimal residual stand, indicating that the 27 in. maximum tree size parameter is not a significant factor.

The equilibrium sustainable diameter distributions which maximize managed forest value and board foot volume growth (for the "zero" cost scenario) are shown in Tables 3 and 4, respectively. A comparison of Tables 2 and 3 shows that maximization of land expectation value and managed forest value do not occur simultaneously. Not only are more big trees present in the latter case, but significantly different levels of residual growing stock (board foot measure) and a slightly different species mixture are also demonstrated. This is principally due to the assessment of interest charges on the value of the residual growing stock when land expectation value is being optimized. Maximization of managed forest value—equiva-

TABLE 2. Equilibrium diameter distribution for maximum land expectation value (interest = 5%; cutting cycle = 10 yr).
Zero Cost Precommercial Thinning

Diameter (in.)	Residual *Picea*	Residual *Abies*	Harvest *Picea*	Harvest *Abies*
0–3	134.13	2.75	0.32	251.04
3–6	42.55	0	0	0.62
6–9	12.21	0	9.22	0.53
9–12	3.38	0	18.14	0
12–15	0.91	0	27.07	0
15–18	0.24	0	0.36	0
18–21	0.06	0	0.08	0
21–24	0.02	0	0.02	0
24–27	0	0	0.01	0
27–30	0	0	0	0
N_s =	193.50	2.75		
B_s =	2.56	0.11		
C_s =	1.05	1.35		

LEV = $255.58/a *MFV* = $442.51/a
VGS = $ 16.08/a *VG* = $170.85/a
BFG = 6696 bf/a *VOL* = 630 bf/a
BA = 14.4 sq.ft./a

Note: *BA* and *VOL* refer to the residual stand basal area and board foot volume, respectively.

TABLE 3. Equilibrium diameter distribution for maximum managed forest value (interest = 5%; Cutting cycle = 10 yr).
Zero Cost Precommercial Thinning

Diameter (in.)	Residual *Picea*	Residual *Abies*	Harvest *Picea*	Harvest *Abies*
0–3	125.40	3.72	0.11	237.08
3–6	37.97	0	0	0.78
6–9	14.90	0	4.65	0.70
9–12	6.30	0	14.23	0
12–15	2.78	0	28.47	0
15–18	1.27	0	0.51	0
18–21	0.59	0	0.25	0
21–24	0.28	0	0.12	0
24–27	0.14	0	0.06	0
27–30	0	0	0.06	0
N_s =	189.63	3.72		
B_s =	2.74	0.22		
C_s =	0.87	2.32		

LEV = $213.43/a *MFV* = $476.62/a
VGS = $ 79.18/a *VG* = $184.02/a
BFG = 6679 bf/a *VOL* = 2066 bf/a
BA = 22.1 sq.ft./a

Note: *BA* and *VOL* refer to the residual stand basal area and board foot volume, respectively.

lent to the classical interest-free forest rent criterion—permits larger (more valuable) trees to occur in the residual distribution.

The maximization of board foot volume growth (Table 4) illustrates the significance of management objectives in stand-level optimization. By removing the effect of species dependent stumpage prices, the optimal solution now consists of a mixture of *Abies* and *Picea*. Yet, for maximum board

TABLE 4. Equilibrium diameter distribution for maximum board foot volume growth (interest = 5%; cutting cycle = 10 yr.).
Zero Cost Precommercial Thinning

Diameter (in.)	Residual *Picea*	Residual *Abies*	Harvest *Picea*	Harvest *Abies*
0–3	22.44	460.94	0.02	0.09
3–6	3.12	118.70	1.94	0.02
6–9	0.70	18.61	2.32	58.91
9–12	0.18	2.35	1.41	69.33
12–15	0.05	0.25	1.43	30.86
15–18	0.02	0.02	0.02	0.07
18–21	0	0	0	0.01
21–24	0	0	0	0
24–27	0	0	0	0
27–30	0	0	0	0
N_s =	26.51	600.87		
B_s =	1.41	2.19		
C_s =	0.83	1.20		

LEV = $201.17/a *MFV* = $333.56/a
VGS = $ 3.61/a *VG* = $128.78/a
BFG = 10383 bf/a *VOL* = 279 bf/a
BA = 29.8 sq.ft./a

Note: *BA* and *VOL* refer to the residual stand basal area and board foot volume, respectively.

foot growth, no tree larger than 18 in. is retained in the residual growing stock over the 10-yr cutting cycle. Hansen and Nyland (1987) also note that large trees are not present in residual diameter distributions that maximize board foot volume growth for uneven-aged stands of sugar maple.

Results for the "true" precommercial thinning scenario produce steady-state stand structures nearly identical to those of the "zero" cost scenario. However, as expected, the cost of the precommercial thinning reduces the land expectation and managed forest values. When maximizing the land expectation value, a periodic precommercial thinning cost of $60.56/ac reduces the land expectation value to $159.71 and the managed forest value to $285.12/ac. When maximizing the managed forest value, a periodic precommercial thinning cost of $62.19/ac results in a land expectation value and managed forest value of $114.78 and $315.55/ac, respectively.

Results shown for the two financial objectives (Tables 2 and 3) have a distinctive common feature: the near absence of *Abies* in the sustainable equilibrium diameter distribution. Several factors combine to produce this result. First, the mortality models in Prognosis indicate that *Abies* possesses a much higher rate of mortality than does *Picea*. Thus, unless harvested, *Abies* does not survive to grow into the larger diameter classes. Second, abundant *Abies* regeneration is assured no matter what the overstory composition. This is due to the recruitment models in Prognosis where neither distance to seed source nor tree size are factors in predicting recruitment success. However, *Picea* regeneration is much more limiting. Thus, cutting in smaller diameter classes must favor *Picea* to ensure sustainability (Alexander 1985). Lastly, species dependent stumpage prices (Table 1) heavily favor *Picea* over *Abies*. The net effect of these three factors is that residual diameter distributions with fewer, larger, and more valuable *Picea* produce more financial gain than stands containing *Abies*.

For the board foot volume growth objective (Table 4), the absence of stumpage prices removes the major price incentive for favoring *Picea* over *Abies*. Coupled with the abundance of *Abies* regeneration and fast *Abies* growth rates in the pole-sized diameter classes, residual stands consist predominately of *Abies*. However, no tree larger than 18 in. is present in the residual stand.

It is also important to point out that the database used to develop the Prognosis regeneration model does not include any true selection harvests. Further, only 3 of 221 sample plots represent selection harvests in the *Abies lasiocarpa/Clintonia uniflora* habitat type (Ferguson et al. 1986). Thus, recruitment estimates are largely based on data from the 709 selection harvest sample plots taken in other habitat types. Under natural regeneration (as assumed here), the recruitment model plays a critically important role in the development of the optimal sustainable equilibrium diameter distribution. Thus, results presented here must be interpreted with this in mind. And, as shown in Tables 2 and 3, it is apparent that *Picea* recruitment is limiting the level of growing stock in the optimal residual distributions.

SUMMARY

Optimizing uneven-aged management decisions has received considerable attention in the past 15 years. However, little work on optimizing the species composition of sustainable equilibrium diameter distributions has been reported. Further, most optimization models have dealt with whole stand and not individual tree-based models.

Using the Prognosis Model—a single tree distance independent growth

model with species dependent growth dynamics—the species mix that maximizes value and volume-based objective functions under a sustainable equilibrium constraint are derived. This is accomplished by converting the stochastic features of Prognosis to their deterministic counterparts. A direct search, derivative-free, constrained nonlinear programming alogrithm is used to optimize this deterministic model for the *Abies lasiocarpa/Clintonia uniflora* habitat type on the Coeur d'Alene National Forest in northern Idaho.

The "investment-efficient" sustainable equilibrium diameter distribution associated with the maximization of the land expectation value differs from the diameter distribution associated with maximization of managed forest value. Further, maximization of board foot volume growth produces diameter distributions dramatically different from either of these financial objectives. These results clearly demonstrate the interrelationship between management objectives and optimal stand structures. By varying the cutting cycle length (in multiples of the ten-year growth projection period) it is also possible to optimize over this parameter.

The methodology employed in this paper can be used to optimize other habitat types included in the Prognosis Model under a variety of input assumptions. Not only will this produce insights into the workings of Prognosis, it will also lead to a better understanding of selection harvesting in uneven-aged forests.

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